





Tree-Adjoining Grammars: Theory and implementation

Day 1

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University of Washington, Seattle

Language modeling with Tree-Adjoining Grammars

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- Why implementation? ⇒ day 3 part 2

As is frequently pointed out but cannot be overemphasized, an important goal of formalization in linguistics is to enable subsequent researchers to see the defects of an analysis as clearly as its merits; only then can progress be made efficiently.

[Dowty 1979:322]

Details of ...

- formal language theory [Hopcroft, Motwani & Ullmann 2006] (ESSLLI 2019 course: https://user.phil.hhu.de/balogh/esslli-2019-course/)
- parsing with mildly context-sensitive formalisms (LCFRS, 2-MCFG, 2-ACG) [Kallmeyer 2010]

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- · complexity of a language
 - ⇒ determined by the weakest formal grammar that generates it
- · expressive power of the formalism
 - ⇒ TAG: The formalism is part of the theory, so let's try to make it both convenient and minimally expressive!

Schedule

Schedule

- Mon: motivation & basic (L)TAG
- Tue: linguistic applications and using (L)TAG: syntax
- Wed:
 - linguistic applications and using (L)TAG: semantics
 - · introduction to grammar engineering and XMG
- Thu: grammar implementation with XMG
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- lecturers:
 - Kata Balogh (balogh@hhu.de)
 - Simon Petitjean (simon.petitjean@uol.de)

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- course page (QR code following):
 - https://spetitjean.github.io/teaching/summer_schools_and_others/ tree_adjoining_grammars_theory_and_implementation_nasslli/



- formal complexity of natural languages \rightarrow gain insights into

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 ⇒ the general structure of natural language

6

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Hypothesis of the adequacy of expressive power

TAG exactly provides the expressive power needed to treat NL.

Expressive power in terms of a specific generative capacity:

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Expressive power in terms of a specific generative capacity:

- weak generative capacity → to generate string languages
- strong generative capacity → to generate tree languages
- derivational generative capacity

Grammar Formalisms

Aim: find an adequate formal system for natural language analysis

- · mathematically concise representation of a grammar theory
- a formal system for linguistic analyses

Grammar Formalisms

Aim: find an adequate formal system for natural language analysis

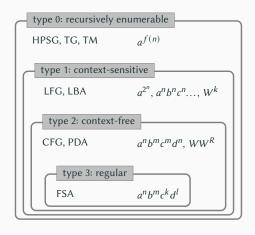
- mathematically concise representation of a grammar theory
- a formal system for linguistic analyses

A formal grammar (N, T, S, R) is

- Type 0 or unrestricted (phrase structure) grammar iff every production is of the form $\alpha \to \beta$ with $\alpha \in (N \cup T)^* \setminus T^*$ and $\beta \in (N \cup T)^*$; generates a recursively enumerable language (RE).
- Type 1 or context-sensitive grammar iff every production is of the form $\gamma A\delta \to \gamma \beta \delta$ with $\gamma, \delta, \beta \in (N \cup T)^*, A \in N$ and $\beta \neq \epsilon$; generates a context-sensitive language (CS).
- Type 2 or context-free grammar iff every production is of the form $A \to \beta$ with $A \in N$ and $\beta \in (N \cup T)^* \setminus \{\epsilon\}$; generates a context-free language (CF).
- Type 3 or right-linear grammar iff every production is of the form $A \to \beta B$ or $A \to \beta$ with $A, B \in N$ and $\beta \in T^* \setminus \{\epsilon\}$; generates a regular language (REG).

How much expressive power do we need to treat NL?

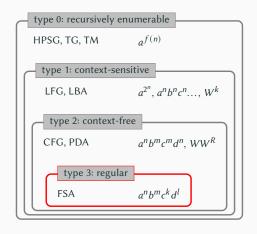
(FSA = finite state automaton, PDA = push-down automaton, EPDA = embedded push-down automaton, LBA = linear bounded automaton, TG = transformational grammar, TM = Turing Machine)



Chomsky(-Schützenberger) hierarchy [Chomsky-Schuetzenberger 1963]

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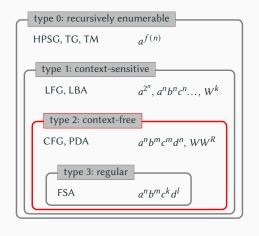
Chomsky(-Schützenberger) hierarchy [Chomsky-Schuetzenberger 1963]

NL is not regular! [Chomsky 1956, 1957] center embedding with relative clauses

n1 n2 n3 v3 v2 v1

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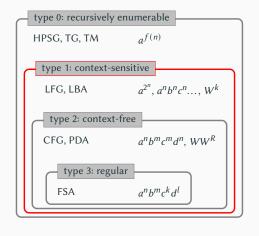


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NL is not context-free! [Shieber 1985] cross serial dependencies in Dutch and Swiss-German

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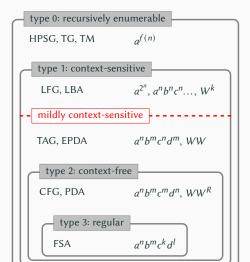


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Is NL context-sensitive?

How much expressive power do we need to treat NL?

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Chomsky(-Schützenberger) hierarchy [Chomsky-Schuetzenberger 1963]

NL is mildly context-sensitive

[Joshi 1985]

- ⊃ CFL
- · cross-serial dep.
- · semi-linear
- in PTIME

Chomsky-hierarchy: overview

Languages as problems:

"Can we decide for every word whether it belongs to L?"

type	grammar	rules	word problem
RE	phrase structure	$\alpha \to \beta$	undecidable
CS	context-sensitive	$\gamma A \delta \rightarrow \gamma \beta \delta$	exponential
CF	context-free	$A \rightarrow \beta$	cubic
REG	right-linear	$A \rightarrow aB b$	linear

For Type 1-3 languages a rule $S \rightarrow \epsilon$ is allowed if S does not occur in any rule's right-hand side.

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 - · only weak-lexicalization possible
- · natural languages are almost context-free

mildly context sensitive languages

 $RL \subset CFL \subset MCSL \subset CSL \subset RE$

[Joshi, 1985]

Mild context sensitivity

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[Joshi, 1985]

 for natural languages we need grammars, that are somewhat richer than context-free grammars, but more restricted than context-sensitive grammars

Mildl context sensitivity

- Joshi (1985): characterize the amount of context-sensitivity needed for NL
- · mildly context sensitive formalisms are such that they
 - · generate at least all CFs
 - can describe a limited amount of cross-serial dependencies (there is an $n \ge 2$ up to which the formalism can generate all string languages $\{w^n|w\in\Sigma^*\}$)
 - · are polynomially parsable
 - their string languages are of constant growth
 (the length of the words generated by the grammar grows in a linear way)

Limits of CFG: expressivity challenge

- German: nested dependency (subordinate clauses)
 - (1) Jan sagte daß er die Kinder dem Hans das Haus streichen helfen ließ. John said that he the children the Hans the house paint help let. 'John said that he let the children to help Hans to paint the house.'



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- Schwyzerdütsch: cross-serial dependency
 - (2) Jan säit das mer d'chind em Hans es huus lönd hälfe aastriiche. John said that we children.acc the Hans.dat the house.acc let help paint. 'John said that we let the children to help Hans to paint the house.'



(3) *mer d'chind de Hans es huus lönd hälfe aastriiche. we children.acc the Hans.acc the house.acc let help paint.

Limits of CFG: expressivity challenge

Proof by Shieber

[Shieber 1985: 334-337]

- series of NPs followed by series of Vs
- raising verb can occur in between
 Jan säit das mer d'chind em Hans es huus lönd hälfe aastriiche.
 - (4) ... mer d'chind em Hans es huus haend wele lönd hälfe aastriiche.
 ... we children.acc the Hans.dat the house.acc have wanted let help paint.
 '... that we have wanted to let the children to help Hans to paint the house.'
- Jan säit das mer NP* es huus haend wele VP* aastriiche
- homomorphism f:

```
f(\operatorname{d'chind}) = a f(\operatorname{em \ Hans}) = b f(\operatorname{l\"ond}) = c f(\operatorname{h\"alfe}) = d f(\operatorname{Jan \ s\"ait \ das \ mer}) = w f(\operatorname{es \ huus \ haend \ wele}) = x f(\operatorname{aastriiche}) = y f(s) = z \operatorname{otherwise}
```

- $f(Schwyzerdütsch) \cap wa^*b^*xc^*d^*y = wa^mb^nxc^md^ny$
 - CFLs are closed under intersection with regular languages: $L1_{CF} \cap L2_{REG} = L3_{CF}$
 - wa*b*xc*d*y is regular
 - by Pumping Lemma: $wa^mb^nxc^md^ny$ is not context-free
- ⇒ Schwyzerdütsch is not context-free

Take a simple CFG

- · string rewriting
- replace non-terminals by strings of terminals and non-terminals

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$$\begin{split} &G_{\mathsf{CFG}} = \langle \mathsf{N}, \mathsf{T}, \mathsf{S}, \mathsf{P} \rangle \\ &\mathsf{P} = \{ \; \mathsf{S} \to \mathsf{NP} \; \mathsf{VP}, \mathsf{VP} \to \mathsf{V} \; \mathsf{NP} \; | \; \mathsf{V}, \mathsf{V} \to \mathit{likes} \; | \; \mathit{like} \; | \; \mathit{sleeps}, \; \mathsf{NP} \to \mathit{she} \; | \; \mathit{her} \; | \; \mathit{they} \; \} \end{split}$$

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Example derivation history:

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$$G_{CFG} = \langle N, T, S, P \rangle$$

P = { S \rightarrow NP VP, VP \rightarrow V NP | V, V \rightarrow likes | like | sleeps, NP \rightarrow she | her | they }

Example derivations:

$$S \rightarrow NP VP \rightarrow she VP \rightarrow she V \rightarrow she sleeps$$

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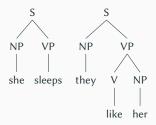
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$$S \rightarrow NP \ VP \rightarrow they \ VP \rightarrow they \ V \ NP \rightarrow they \ like \ NP \rightarrow they \ like \ her$$

Example derivation history:



- subcategorization / argument selection
 - (1) She sleeps. / She likes her. / *She likes.

$$S \Rightarrow NP VP \Rightarrow Joe VP \Rightarrow Joe V \Rightarrow Joe sleeps$$

$$S {\Rightarrow} \; \mathsf{NP} \; \mathsf{VP} {\Rightarrow} \; \mathsf{Joe} \; \mathsf{VP} {\Rightarrow} \; \mathsf{Joe} \; \mathsf{V} {\Rightarrow} \; \mathsf{Joe} \; \mathsf{likes}$$

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- encode necessary information in the non-terminals?

S
$$\rightarrow$$
 NP_{3sg/nom} VP_{3sg/itr}, S \rightarrow NP_{3pl/nom} VP_{3pl/itr},
S \rightarrow NP_{3sg/nom} VP_{3sg/tr}, S \rightarrow NP_{3pl/nom} VP_{3pl/tr},
VP_{3sg/tr} \rightarrow V_{3sg/tr} NP_{3sg/acc}, VP_{3pl/tr} \rightarrow V_{3pl/tr} NP_{3sg/acc},
VP_{3sg/itr} \rightarrow V_{3sg/itr}, VP_{3pl/itr} \rightarrow V_{3pl/itr},
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extend for number agreement, argument selection (transitive vs. non-transitive) and case marking

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every possible combination of arguments selection (e.g. transitive/non-transitive), number agreement and case marking must have a separate non-terminal and a separate re-write rule

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V<sub>3sg/itr</sub> \rightarrow sleeps, V<sub>3pl/itr</sub> \rightarrow sleep, V<sub>3sg/tr</sub> \rightarrow likes, V<sub>3pl/tr</sub> \rightarrow like
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- solution: feature structures, unification, underspecification (see later)

Lexicalized grammar

A lexicalized grammar consists of:

- (i) a finite set of structures each associated with a lexical item (anchor),
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weak vs. strong lexicalization

- · weak lexicalization: preserve the string language
- · strong lexicalization: preserve the tree structure

Linguistically interesting:

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Computationally interesting:

- the search space during parsing can be delimited (grammar filtering)
- · use of corpora in NLP

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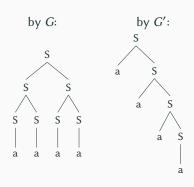
Can CFGs be strongly lexicalized (= the set of trees are preserved)?

Answer:

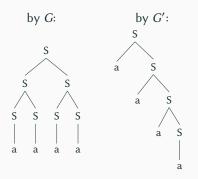
No. Only weak lexicalization possible (= same string language).

- example:
 - a CFG $G: S \rightarrow SS, S \rightarrow a$
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G cannot be strongly lexicalized with some finite CFG, e.g., G'.

From CFG to TAG: Tree Substitution Grammar (TSG)

- · a CFG rule corresponds to a tree
 - LHS as the root node / RHS as the daughter nodes
 - e.g., $S \rightarrow NP VP$



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A TSG is a quadruple $TSG = \langle \Sigma, N, S, I \rangle$, where

 Σ is a set of terminal symbols;

N is a set of non-terminal symbols;

 $S \in N$ is a distinguished non-terminal symbol;

I is a finite set of initial trees.

$$G_{CFG} = \langle N, T, S, P \rangle$$

$$P = \{$$

$$S \rightarrow NP \ VP$$

$$VP \rightarrow V \ NP \ | \ AP \ VP$$

$$NP \rightarrow N \ | \ Det \ N$$

$$AP \rightarrow A$$

$$N \rightarrow Peter \ | \ fridge$$

$$Det \rightarrow the$$

$$A \rightarrow easily$$

$$V \rightarrow repaired$$

$$Peter \ fridge \ the \ repaired$$

$$Peter \ fridge \ the \ repaired$$

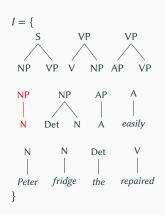
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the

Peter

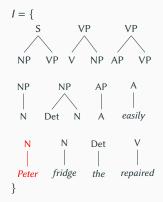
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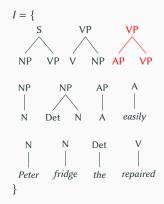


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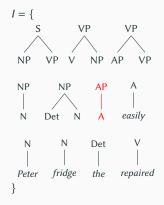


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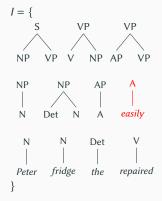


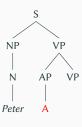
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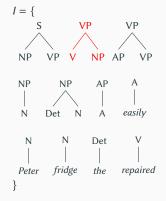


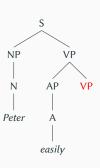
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VP

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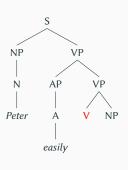
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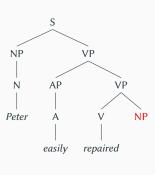


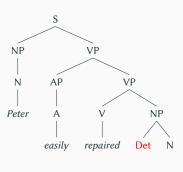
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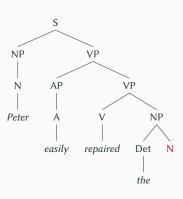
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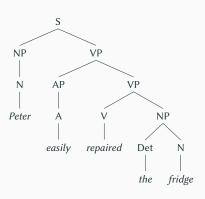
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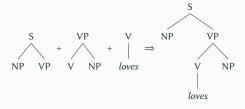




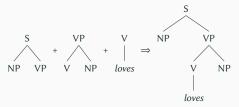
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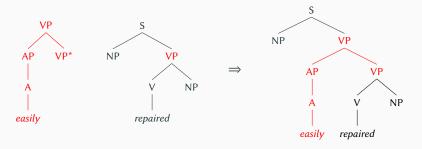
- · Some applications of TSG:
 - in data-oriented parsing (DOP) (Bod 1995),
 - Lexicalized TSGs can be extracted from treebanks and used for probabilistic parsing (Post & Gildea 2009).

• lexicalization of CFG in a linguistically meaningful way

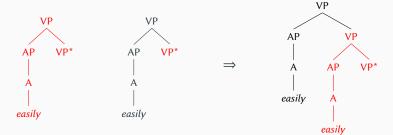
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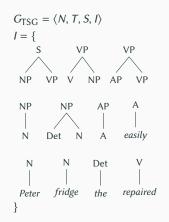
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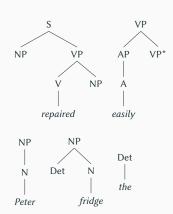


- ⇒ Adjunction at footnodes causes spurious ambiguities in derivations.
- \Rightarrow Therefore, this is usually forbidden.

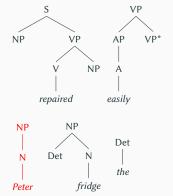


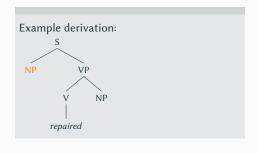
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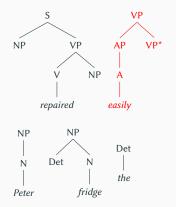


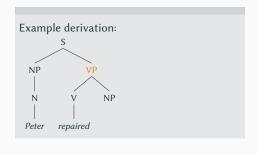
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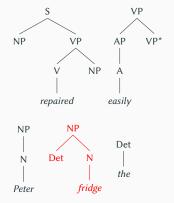


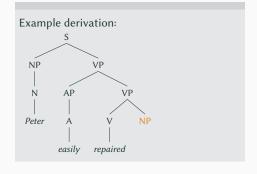
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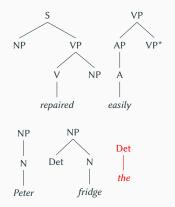


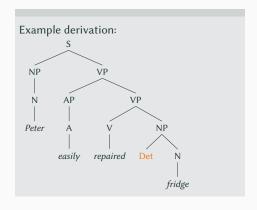
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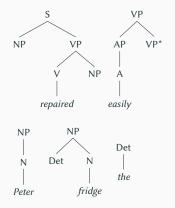


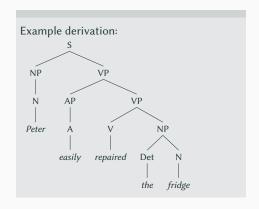
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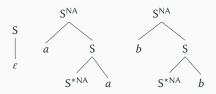
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Adjunction constraints are essential in generating non-context-free languages (e.g., the copy language $\{ww|w\in\{a,b\}^*\}$)!

From CFG to TAG: Restrictions on adjunction

Example grammar for the copy language $\{ww|w \in \{a,b\}^*\}$:



 \Rightarrow TAG = TSG + adjunction + adjunction constraints

Example: derivation of *abbabb*

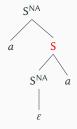
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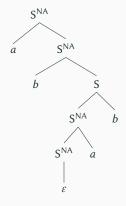


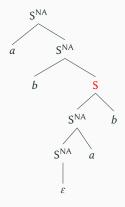
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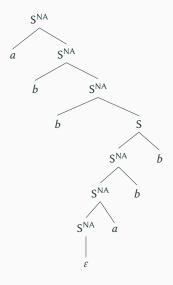












Tree Adjoining Grammar (TAG)

A Tree Adjoining Grammar is a tuple $G = \langle N, T, I, A, O, C \rangle$:

T and N are disjoint alphabets of terminals (T) and non-terminals (N),

I is a finite set of **initial trees**, and

A is a finite set of **auxiliary trees**.

 $O: \{v \mid v \text{ is a node in a tree in } I \cup A\} \rightarrow \{1, 0\} \text{ is a function, and}$

 $C: \{v \mid v \text{ is a node in a tree in } I \cup A\} \rightarrow \mathcal{P}(A) \text{ is a function.}$

The trees in $I \cup A$ are called **elementary trees**.

Let v be a node in $I \cup A$:

- obligatory adjunction (OA): O(v) = 1
- null adjunction (NA): O(v) = 0 and $C(v) = \emptyset$
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TAG is mildly context-sensitive (MCS; Joshi 1985)

- generates the context-free languages
- generates cross-serial dependencies (i.e. ww)
- constant growth (or semi linear, no a^{2^n})
- polynomial time parsing $(O(n^6))$

[Schabes 1990, Joshi & Schabes 1997, Kallmeyer: 2010]

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⇒ expressivity challenge ✓

TAG can **strongly lexicalize** finitely ambiguous CFG.

[Schabes 1990, Joshi & Schabes 1997]

(formally, computationally and linguistically interesting (see slide 17))

⇒ lexicalization ✓

• linguistically adequate:

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 - · Phenomena:

linearization, agreement, discontinuity, ellipsis, coordination, ...

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 - · correct predictions wrt. processing complexity

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